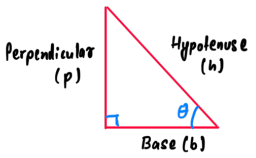
Trigonometry - Class X by Sushil Sir

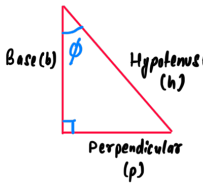


$\sin\theta = \frac{p}{h}$ Reciprocal $\rightarrow$ $\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{h}{p}$

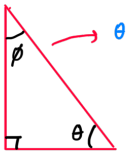
$\cos\theta = \frac{b}{h}$ Reciprocal $\rightarrow$ $\sec\theta = \frac{1}{\cos\theta} = \frac{h}{b}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{p}{b}$ Reciprocal $\rightarrow$ $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} = \frac{b}{p}$

- $\sin\theta \operatorname{cosec}\theta = 1$
- $\cos\theta \sec\theta = 1$
- $\tan\theta \cot\theta = 1$



$\sin\phi = \frac{p}{h}$, $\cos\phi = \frac{b}{h}$, $\tan\phi = \frac{p}{b}$



$\theta + \phi = 90^\circ$ [Complementary]

$\sin\phi = \cos\theta$	$\sec\phi = \operatorname{cosec}\theta$
$\cos\phi = \sin\theta$	$\operatorname{cosec}\phi = \sec\theta$
$\tan\phi = \cot\theta$	$\cot\phi = \tan\theta$

Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$

$1 - \sin^2\theta = \cos^2\theta$ $\qquad$ $1 - \cos^2\theta = \sin^2\theta$

2. $1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$ $\qquad$ $\sec^2\theta - 1 = \tan^2\theta$

Apply: $a^2 - b^2 = (a-b)(a+b)$

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$ V. imp...

3. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$ $\qquad$ $\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

Apply: $a^2 - b^2 = (a-b)(a+b)$

$(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta) = 1$

$\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$ V. Imp..

Standard Angle

	0°	30°	45°	60°	90°
$\sin\theta$	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} = 1$
$\cos\theta$	$\sqrt{\frac{4}{4}} = 1$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{0}{4}} = 0$
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞ (Not defined)

$-1 \le \sin\theta \le 1$

$-1 \le \cos\theta \le 1$

Important Points

- $\sin\theta - \cos\theta = 0 \implies \tan\theta = 1 \implies \theta = 45^\circ$
- For any $x > 0$, $x + \frac{1}{x} \ge 2 \implies \cos x + \frac{1}{\cos x} \ge 2$
- $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 179^\circ \cos 180^\circ = 0$
- If $\tan\theta + \cot\theta = 2$, then $\tan^n\theta + \cot^n\theta = 2$
- If $\sec\theta + \tan\theta = x$, then $\sec\theta - \tan\theta = \frac{1}{x}$, $\sec\theta = \frac{x^2+1}{2x}$, $\tan\theta = \frac{x^2-1}{2x}$
- Prove that $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

Step 1: Divide by $\sin A$ both Nr & Dr.

Step 2: $1 \rightarrow \operatorname{cosec}^2 A - \cot^2 A = (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)$

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Given: $PR - PQ = x$ $\quad -(i)$

Pythagoras Theorem

$PR^2 = PQ^2 + RQ^2$

$RQ^2 = PR^2 - PQ^2 = (PR - PQ)(PR + PQ)$

$PR + PQ = \frac{RQ^2}{PR - PQ} = \frac{b^2}{x}$ $\quad -(ii)$