Real Numbers - Class X by Sushil Sir

- **Prime Numbers** → only two factors → 1 and number itself
  - 2, 3, 5, ... → 2 is only even prime no.
  - → 3 is smallest odd prime no.
- **Composite Numbers** → more than two factors
  - 4, 6, 8, ... → 9 is smallest odd composite no.
- **1 is neither prime nor composite.**
- For any two positive integers a and b.

$$HCF(a,b) \times LCM(a,b) = a \times b$$

or $HCF(a,b) = \frac{a \times b}{LCM(a,b)}$ or $LCM(a,b) = \frac{a \times b}{HCF(a,b)}$

- **Co-prime numbers:** $HCF(a,b) = 1$ e.g. $HCF(2,3) = 1$
  - Consecutive number is co-prime.
- $N_1 = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \cdots p_n^{a_n}$

  $N_2 = p_1^{b_1} \cdot p_2^{b_2} \cdot p_3^{b_3} \cdots p_n^{b_n}$

  $HCF(N_1, N_2) = p_1^{\min(a_1,b_1)} \cdot p_2^{\min(a_2,b_2)} \cdots p_n^{\min(a_n,b_n)}$

  $LCM(N_1, N_2) = p_1^{\max(a_1,b_1)} \cdot p_2^{\max(a_2,b_2)} \cdots p_n^{\max(a_n,b_n)}$

e.g. $N_1 = 2^3 \cdot 3^4 \cdot 5^1$, $N_2 = 2^1 \cdot 3^2 \cdot 5^2$

$HCF(N_1, N_2) = 2^{\min(3,1)} \cdot 3^{\min(4,2)} \cdot 5^{\min(1,2)} = 2^1 \cdot 3^2 \cdot 5^1$

$LCM(N_1, N_2) = 2^{\max(3,1)} \cdot 3^{\max(2,4)} \cdot 5^{\max(1,2)} = 2^3 \cdot 3^4 \cdot 5^2$

- **Fundamental Theorem of Arithmetic**
  - Composite Number = Product of prime numbers
- $p \mid a^2$ then $p \mid a$ where p is a prime number.
  - e.g. $3 \mid 9$ then $3 \mid 3$
- Prove $\sqrt{n}$ is irrational. E.g. $\sqrt{2}, \sqrt{3}, \sqrt{7}, \sqrt{5}, \ldots$

  Let $\sqrt{n} = \frac{p}{q}$, $HCF(p,q) = 1$

  $nq^2 = p^2 \Rightarrow n \mid p^2 \Rightarrow n \mid p \Rightarrow p = nc$

  $\Rightarrow nq^2 = n^2c^2 \Rightarrow q^2 = nc^2 \Rightarrow n \mid q^2 \Rightarrow n \mid q \Rightarrow q = nc$

  $HCF(p,q) = n$ → Contradicts

  Hence, $\sqrt{n}$ is irrational no.
- **Decimal**
  - Terminating → Rational
  - Non-terminating
    - Recurring (Rational) e.g. $1.4\overline{6}$
    - Non-recurring (Irrational) 1.41621432121...
- $\sqrt{2} = 1.41421$, $\sqrt{3} = 1.7320$, $\sqrt{5} = 2.2360$