Polynomials - Class X by Sushil Sir

- For an algebraic expression to be a polynomial
  - ↳ exponent of variable should be non-negative integral power.
  - e.g. $x+x^{-1}$, $\sqrt{x}$, $2x\sqrt{x}$, $x+\frac{1}{x}$, $x^{1/4}$, $x^2+\frac{1}{x^2}$ } Not a polynomial.

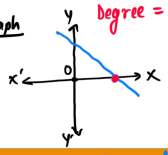
- $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1 x + a_0$ is called a polynomial in variable x.

- Degree → exponent of a highest degree term.
  - Not defined → Zero Polynomial $f(x)=0$
  - Zero → Constant Polynomial $f(x)=k$
  - One → Linear polynomial $f(x)=ax+b,\ a\neq 0$
  - Two → Quadratic Polynomial $f(x)=ax^2+bx+c,\ a\neq 0$
  - Three → Cubic Polynomial $f(x)=ax^3+bx^2+cx+d,\ a\neq 0$

- Degree = n → it means a polynomial can have atmost n real zeros.
  - ↳ it also means how many times it will touch x-axis.

- Linear Polynomial
  $f(x)=ax+b,\ x\neq 0$

  Graph

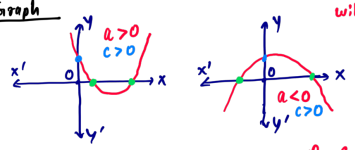
  

  Degree = 1 → No. of zeros
  → No. of times touches x-axis

- Quadratic Polynomial
  $f(x)=ax^2+bx+c,\ a\neq 0$

  Degree = 2 → No. of zeros → α, β
  → No. of times it will touch x-axis.

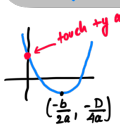
  Graph

  a → Coefficient of x^2
  b → Coefficient of x
  c → Constant term

  # → Concave up → $a>0$

  # → Concave down → $a<0$

  # 

  → $c>0$

  # 

  → $c<0$

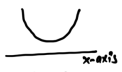
  # Discriminant, $D=b^2-4ac$
  ↳ Nature of roots

  $D>0$
  Unique / distinct roots

  $D=0$
  equal roots

  $D<0$
  Imaginary roots

- $f(x) = ax^2+bx+c = (x-\alpha)(x-\beta)$
  where α and β are roots.

  $S = \alpha+\beta = -\frac{b}{a}$
  $P = \alpha\beta = \frac{c}{a}$

  $x^2 - Sx + P = x^2 - (\alpha+\beta)x + \alpha\beta$

- $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
  $\alpha-\beta = \frac{\sqrt{D}}{a} = \frac{\sqrt{b^2-4ac}}{a}$