



MATHEMATICS TRICKS & TIPS

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1. ALGEBRA TOOLBOX

- $a^m \times a^n = a^{(m+n)}$; $a^m/a^n = a^{(m-n)}$, for $a \neq 0$.
- $\log_b(xy) = \log_b x + \log_b y$; $\log_b(x^k) = k \log_b x$.
- For $\log_b x$: require $x > 0$, $b > 0$ and $b \neq 1$. Check the domain first.
- $(a+b)^2 = a^2 + 2ab + b^2$; $a^2 - b^2 = (a-b)(a+b)$.
- Rationalise a root expression by multiplying numerator and denominator by its conjugate.
- For non-negative a, b : AM-GM says $(a+b)/2 \geq \sqrt{ab}$; equality occurs at $a=b$.

2. QUADRATIC EQUATIONS

- For $ax^2+bx+c=0$: sum of roots = $-b/a$; product = c/a .
- Discriminant $D = b^2 - 4ac$. $D > 0$: distinct real roots; $D=0$: equal; $D < 0$: non-real pair.
- If roots are α and β , the monic equation is $x^2 - (\alpha+\beta)x + \alpha\beta = 0$.
- Vertex of $y=ax^2+bx+c$ occurs at $x = -b/(2a)$; vertex $y = -D/(4a)$.
- For $a > 0$, the parabola opens upward; for $a < 0$, it opens downward.
- In inequalities, find critical points, test one value per interval, and include endpoints only if allowed.

3. SEQUENCES & SERIES

- AP: $a_n = a + (n-1)d$; $S_n = n/2 [2a + (n-1)d]$.
- GP: $a_n = ar^{(n-1)}$; $S_n = a(r^n - 1)/(r - 1)$, $r \neq 1$.
- Infinite GP: $S = a/(1-r)$, only when $|r| < 1$.
- If A and G are the AM and GM of positive x, y , then $A \geq G$.
- Sum 1 to $n = n(n+1)/2$; sum of squares = $n(n+1)(2n+1)/6$.
- Before using an AP/GP formula, identify the first term, common difference/ratio and number of terms.

4. BINOMIAL THEOREM

- General term: $T_{(r+1)} = C(n, r) a^{(n-r)} b^r$ in $(a+b)^n$.
- $C(n, r) = n!/[r!(n-r)!] = C(n, n-r)$.
- For n even there is one middle term; for n odd there are two middle terms.
- Sum of all coefficients in $(1+x)^n = 2^n$.
- Sum of alternating coefficients in $(1+x)^n$ is 0 for $n \geq 1$, by putting $x = -1$.
- For a requested power of x , match its exponent in the general term before solving for r .

5. COMPLEX NUMBERS

- $z = a+ib$; conjugate $\bar{z} = a-ib$; $|z| = \sqrt{a^2+b^2}$.
- $z \times \bar{z} = |z|^2 = a^2+b^2$.
- Powers of i repeat every 4: $i, -1, -i, 1$. Reduce the exponent modulo 4.
- Distance on the Argand plane: distance between $z_1, z_2 = |z_1 - z_2|$.
- Real part: $(z+\bar{z})/2$; imaginary part: $(z-\bar{z})/(2i)$.
- For real-coefficient polynomials, non-real roots occur in conjugate pairs.

6. PERMUTATION & COMBINATION

- $nPr = n!/(n-r)!$; $nCr = n!/[r!(n-r)!]$.
- Permutation = arrangement (order matters); combination = selection (order does not matter).
- Arrange a block of linked objects together first, then treat that block as one object.
- For circular arrangements of n distinct people, rotations are the same: $(n-1)!$ ways.
- At least one = total cases - cases with none.
- For selection with repetition allowed: number of choices of r from n types is $C(n+r-1, r)$.



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1. SETS, FUNCTIONS & DOMAIN

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- For $f(g(x))$, apply g first, then f . Composition order matters.
- An inverse function exists on the stated domain when f is one-one; for inverse onto the codomain too, f must be bijective.
- For $\sqrt{g(x)}$, require $g(x) \geq 0$; for $\log(g(x))$, require $g(x) > 0$.
- For a rational expression, exclude values making the denominator zero.
- Even: $f(-x) = f(x)$; odd: $f(-x) = -f(x)$. Check only when the domain is symmetric about 0.

2. TRIGONOMETRY SHORTCUTS

- $\sin^2 x + \cos^2 x = 1$; $1 + \tan^2 x = \sec^2 x$.
- $\sin(A+B) = \sin A \cos B + \cos A \sin B$.
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$.
- $\sin 2x = 2 \sin x \cos x$; $\cos 2x = \cos^2 x - \sin^2 x$.
- max/min of $a \sin x + b \cos x = \pm \sqrt{a^2 + b^2}$.
- Convert degrees to radians before using standard limits or calculus: $180 \text{ degrees} = \pi \text{ radians}$.

3. STRAIGHT LINES & CIRCLES

- Line through (x_1, y_1) : $y - y_1 = m(x - x_1)$.
- Distance from (x_1, y_1) to $Ax + By + C = 0$ is $\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.
- Parallel lines have equal slopes (when both slopes are defined); perpendicular slopes multiply to -1.
- Circle: $(x-h)^2 + (y-k)^2 = r^2$; centre (h, k) , radius r .
- $x^2 + y^2 + 2gx + 2fy + c = 0$ has centre $(-g, -f)$, radius $\sqrt{g^2 + f^2 - c}$.
- For a tangent at (x_1, y_1) on $x^2 + y^2 = r^2$: $xx_1 + yy_1 = r^2$.

4. CONIC SECTIONS

- Parabola $y^2 = 4ax$: focus $(a, 0)$, directrix $x = -a$; latus rectum length = $4a$.
- Ellipse $x^2/a^2 + y^2/b^2 = 1$, $a > b$: $c^2 = a^2 - b^2$; eccentricity $e = c/a$.
- Hyperbola $x^2/a^2 - y^2/b^2 = 1$: $c^2 = a^2 + b^2$; $e = c/a$.
- For $x^2 = 4ay$, the focus is $(0, a)$ and directrix is $y = -a$.
- Identify the standard form first; do not mix ellipse and hyperbola signs.
- For shifted conics, complete the square in x and y to find the centre or vertex.

5. VECTORS & 3D GEOMETRY

- $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$; dot product = 0 for perpendicular non-zero vectors.
- $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin(\theta)$, equal to parallelogram area.
- Triangle area from vectors = half of $|\mathbf{a} \times \mathbf{b}|$.
- Direction cosines: $l^2 + m^2 + n^2 = 1$.
- 3D distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
- Scalar triple product absolute value gives parallelepiped volume; zero means coplanar vectors.

6. MATRICES & DETERMINANTS

- For $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, determinant = $ad - bc$.
- $\mathbf{A}^{-1} = \text{adj}(\mathbf{A}) / \det(\mathbf{A})$, only when $\det(\mathbf{A}) \neq 0$.
- If $\det(\mathbf{A}) = 0$, the square matrix is singular and has no inverse.
- $\det(\mathbf{AB}) = \det(\mathbf{A}) \det(\mathbf{B})$ for square matrices of the same order.
- For a 2×2 inverse, swap the diagonal entries and change signs of the off-diagonal entries, then divide by determinant.
- For triangle area using coordinates, take half the absolute value of the 3×3 determinant.



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1. LIMITS & CONTINUITY

- Standard limits (x in radians): $\sin x/x \rightarrow 1$; $(e^x - 1)/x \rightarrow 1$.
- $\ln(1+x)/x \rightarrow 1$; $(1+x)^{1/x} \rightarrow e$ as $x \rightarrow 0$.
- If substitution gives $0/0$, try factoring, conjugate multiplication or a standard limit.
- Continuity at $x=a$ requires the left limit = right limit = $f(a)$.
- For limits at infinity, compare highest powers; for rational polynomials degrees decide the leading behaviour.
- Do not use $\sin x / x \rightarrow 1$ if x is in degrees; calculus limits use radians.

2. DIFFERENTIATION & TANGENTS

- $d(x^n)/dx = n x^{(n-1)}$; $d(e^x)/dx = e^x$; $d(\ln x)/dx = 1/x$.
- Product: $(uv)' = u'v + uv'$; quotient: $(u/v)' = (u'v - uv')/v^2$.
- Chain rule: $d[f(g(x))]/dx = f'(g(x)) g'(x)$.
- $d(\sin x) = \cos x$; $d(\cos x) = -\sin x$; $d(\tan x) = \sec^2 x$.
- Tangent at $x=a$: $y - f(a) = f'(a)(x - a)$.
- For maxima/minima, solve $f' = 0$ and test sign change or use the second derivative carefully.

3. INTEGRATION METHODS

- Integral of $x^n dx = x^{(n+1)}/(n+1) + C$, for $n \neq -1$.
- Integral of $1/x dx = \ln|x| + C$; integral of $e^x dx = e^x + C$.
- By parts: integral $u dv = uv - \text{integral } v du$.
- Substitution: choose an inner function whose derivative also appears in the integrand.
- Use partial fractions for rational functions after factoring the denominator (when possible).
- Always add $+C$ for indefinite integrals; differentiate your answer to check it.

4. DEFINITE INTEGRALS & ODE

- For odd f : integral from $-a$ to a of $f(x) dx = 0$; for even f it is 2 times integral 0 to a .
- Symmetry: integral 0 to a of $f(x) dx$ equals integral 0 to a of $f(a-x) dx$.
- Area between curves = integral (upper curve - lower curve) dx between intersection points.
- Separable ODE $dy/dx = g(x)h(y)$: integrate $dy/h(y) = \text{integral } g(x) dx$.
- Find intersection limits before area calculations; split the interval if upper/lower curves switch.
- For differential equations, keep the integration constant and use initial conditions when given.

5. PROBABILITY & STATISTICS

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- $P(A|B) = P(A \cap B)/P(B)$, with $P(B) > 0$.
- For independent events: $P(A \cap B) = P(A)P(B)$. Do not assume independence.
- Bayes: $P(A|B) = P(B|A)P(A)/P(B)$, when $P(B) > 0$.
- Mean = $\sum(fx)/\sum(f)$; variance = $E(X^2) - [E(X)]^2$.
- For counting-based probability, define the sample space and count outcomes consistently.

6. JEE SOLVING STRATEGY

- First identify the chapter and the target: value, number of roots, range, maximum, area or probability.
- Write the relevant condition before calculating: domain, non-zero denominator, positive log argument or parameter range.
- Use options to test special values only when logically valid; one example alone rarely proves an identity.
- Check signs, endpoints, extraneous roots and units/angle mode before choosing an answer.
- If stuck, mark the step where progress stops, move on, and return after easier questions.
- After a mock test, sort errors into concept, formula, algebra, reading and time-management mistakes.