

JEE MAIN – TEST PAPER

●●●●●● General Instructions: ●●●●●●

The test consists of total 75 questions.

Each subject (PCM) has 25 questions.

This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics.

Each part has only two sections: Section-A and Section-B. Section – A : Attempt all questions.
Section – B : Numerical-based questions.

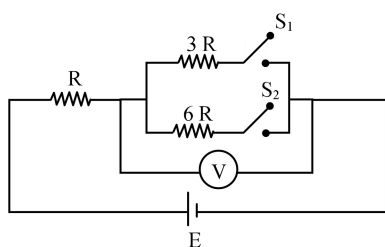
Maximum Marks: 300. Each question carries +4 marks for correct answer and –1 mark for wrong answer.

PART A: PHYSICS (SECTION A)

1. Which of the following groups has different dimensions ?

- (A) Potential difference, emf, voltage (B) Pressure, stress, Young's modulus
(C) Heat, energy, work done (D) Dipole moment, electric flux, electric field

2. In the circuit shown in the figure, reading of voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed and reading of voltmeter is V_3 when both S_1 and S_2 are closed. Then –



- (A) $V_3 > V_2 > V_1$ (B) $V_2 > V_1 > V_3$ (C) $V_3 > V_1 > V_2$ (D) $V_1 > V_2 > V_3$

3. A student performs an experiment to determine the Young's modulus of a wire exactly 2 cm long, by Searle's method. In a particular reading, the student measures the extension in the length of the wire to be 0.8 mm with an uncertainty of ± 0.05 mm at a load of exactly 1.0 kg. The

student also measures the diameter of the wire to be 0.4 mm with an uncertainty of ± 0.01 mm. Take $g = 9.8 \text{ m/s}^2$ (exact).

The Young's modulus obtained from the reading is:

- (A) $(2.0 \pm 0.3) \times 10^{11} \text{ N/m}^2$ (B) $(2.0 \pm 0.2) \times 10^{11} \text{ N/m}^2$
 (C) $(2.0 \pm 0.1) \times 10^{11} \text{ N/m}^2$ (D) $(2.0 \pm 0.05) \times 10^{11} \text{ N/m}^2$

4. A thick spherical shell has inner radius b and outer radius a . The material has resistivity ρ . It's resistance between inner and outer surface is -

- (A) $\frac{\rho(b-a)}{4\pi ab}$ (B) $\frac{\rho(b-a)}{4ab}$ (C) $\frac{\rho(a-b)}{4ab}$ (D) $\frac{\rho(a-b)}{4\pi ab}$

5. An electron of mass m and charge e initially at rest gets accelerated by a constant electric field E . The negative rate of change of De-Broglie wavelength of this electron at time t is -

- (A) $\frac{2h}{eEt^2}$ (B) $-\frac{2h}{eEt^2}$ (C) $\frac{h}{eEt^2}$ (D) $-\frac{h}{eEt^2}$

6. The Bohr radius of an atom of nuclear charge Z is of order (Note : α is the fine structure constant $e^2/\hbar c$.) -

- (A) $\frac{\hbar}{Z\alpha mc}$ (B) $\frac{Z\alpha\hbar}{mc}$ (C) $\frac{mc}{Z\alpha\hbar}$ (D) $\frac{m\alpha}{Z\hbar c}$

7. Consider a solid cube of uniform charge density of insulating material. What is the ratio of the electrostatic potential at a corner to that at the centre. (Take the potential to be zero at infinity, as usual)

- (A) $\frac{1}{1}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{9}$

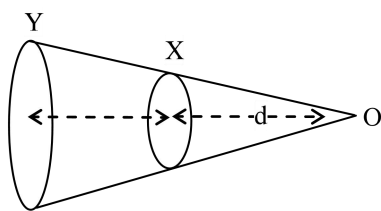
8. A uniformly charged and infinitely long line having a linear charge density ' λ ' is placed at a normal distance y from a point O . Consider a sphere of radius R with O as centre and $R > y$. Electric flux through the surface of the sphere is-

- (A) zero (B) $\frac{2\lambda R}{\epsilon_0}$ (C) $\frac{2\lambda\sqrt{R^2 - y^2}}{\epsilon_0}$ (D) $\frac{\lambda\sqrt{R^2 + y^2}}{\epsilon_0}$

9. Two metallic charged sphere whose radii are 20 cm and 10 cm respectively, each having 150 μC positive charge. The common potential after a conducting wire connects them is -

- (A) $9 \times 10^6 \text{ V}$ (B) $4.5 \times 10^6 \text{ V}$ (C) $1.8 \times 10^7 \text{ V}$ (D) $13.6 \times 10^6 \text{ V}$

10. Two circular coils X and Y have equal number of turn and carry equal currents in the same sense and subtend same solid angle at point O . If the smaller coil X is midway between O and Y , then if we represent the magnetic induction due to bigger coil Y at O as B_Y and that due to smaller coil X at O B_X , then :



- (A) $\frac{B_Y}{B_X} = 1$ (B) $\frac{B_Y}{B_X} = 2$ (C) $\frac{B_Y}{B_X} = \frac{1}{2}$ (D) $\frac{B_Y}{B_X} = \frac{1}{4}$

11. A rectangular loop of metallic wire is of length a and breadth b and carries a current i . The magnetic field at the centre of the loop is -

- (A) $\frac{\mu_0 i}{4\pi} \frac{8\sqrt{a^2 + b^2}}{ab}$ (B) $\frac{\mu_0 i}{4\pi} \frac{4\sqrt{a^2 + b^2}}{ab}$
 (C) $\frac{\mu_0 i}{4\pi} \frac{2\sqrt{a^2 + b^2}}{ab}$ (D) $\frac{\mu_0 i}{4\pi} \frac{\sqrt{a^2 + b^2}}{ab}$

12. An AC generator of 220V having internal resistance $r = 10\Omega$ and external resistance $R = 100\Omega$. What is the power developed in the external circuit?

- (A) 484 W (B) 400 W (C) 441 W (D) 369 W

13. A plane electromagnetic wave of wavelength λ has an intensity I . It is propagating along the positive Y- direction. The allowed expressions for the electric and magnetic fields are given by

- (A) $\vec{E} = \sqrt{\frac{I}{\epsilon_0 C}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right] \hat{i}; \vec{B} = \frac{1}{c} E \hat{k}$
 (B) $\vec{E} = \sqrt{\frac{I}{\epsilon_0 C}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right] \hat{k}; \vec{B} = -\frac{1}{c} E \hat{i}$
 (C) $\vec{E} = \sqrt{\frac{2I}{\epsilon_0 C}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right] \hat{k}; \vec{B} = +\frac{1}{c} E \hat{i}$
 (D) $\vec{E} = \sqrt{\frac{2I}{\epsilon_0 C}} \cos\left[\frac{2\pi}{\lambda}(y + ct)\right] \hat{k}; \vec{B} = \frac{1}{c} E \hat{j}$

14. The relation $3t = \sqrt{3x} + 6$ describes the displacement of a particle in one direction where x is in metres and t in sec. The displacement, when velocity is zero, is

- (A) 24 metres (B) 12 metres (C) 5 metres (D) Zero

15. A spring has a force constant k and mass m . The spring hangs vertically and a block of unknown mass is attached to its bottom end: It is known that the mass of the block is much greater than that of the spring. The hanging block stretches the spring the twice its relaxed length.

How long (t) would it take for a low amplitude transverse pulse to travel the length of the spring stretched by the hanging block ?

- (A) $\sqrt{\frac{2m}{k}}$ (B) $\sqrt{\frac{m}{k}}$ (C) $\sqrt{\frac{m}{2k}}$ (D) $\sqrt{\frac{2m}{3k}}$

16. A particle at the end of a spring executes simple harmonic motion with a period t_1 , while the corresponding period for another spring is t_2 . If the period of oscillation with the two springs in series is T, then -

- (A) $T = t_1 + t_2$ (B) $T^2 = t_1^2 + t_2^2$ (C) $T^{-1} = t_1^{-1} + t_2^{-1}$ (D) $T^{-2} = t_1^{-2} + t_2^{-2}$

17. The equation for the displacement of a stretched string is given by :

$$y = 4 \sin 2\pi \left(\frac{t}{0.02} - \frac{x}{100} \right)$$

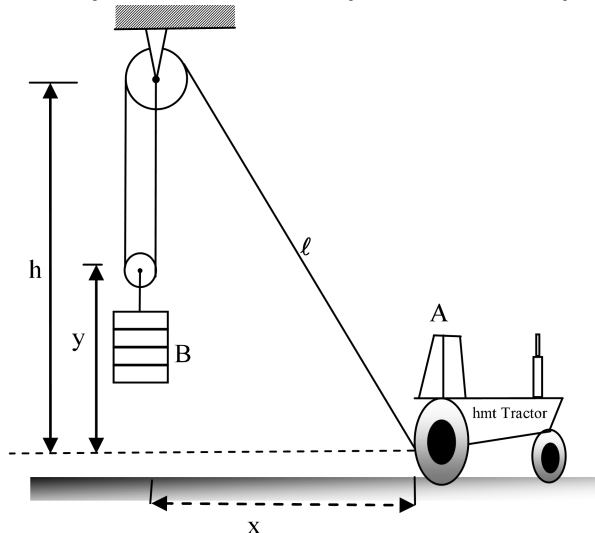
Where y and x are in cm and t in sec. The

- (i) frequency (ii) velocity of the wave
(iii) maximum particle velocity are -

- (A) 50 Hz, 50 m/s, 20π m/s (B) 50 Hz, 20 m/s, 50 m/s
(C) 50 Hz, 50 m/s, 2π m/s (D) 50 Hz, 50 m/s, 4π m/s

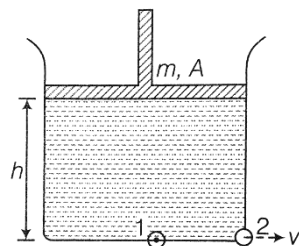
18.

The tractor A is used to hoist the bale B with the pulley arrangement shown. If A has a forward velocity v_A , determine an expression for the upward velocity v_B of the bale in terms of x.



- (A) $\frac{v_A x}{h}$ (B) $\frac{1}{2} \frac{v_A x}{\sqrt{h^2 + x^2}}$ (C) $\frac{1}{2} \frac{v_A h}{\sqrt{h^2 + x^2}}$ (D) $\frac{v_A h}{x}$

19. A cylindrical vessel contains a liquid of density ρ upto a height h . The liquid is closed by a piston of mass m and area of cross-section A . There is a small hole at the bottom of the vessel. The speed v with which the liquid comes out of the hole is



- (A) $\sqrt{2gh}$ (B) $\sqrt{2\left(gh + \frac{mg}{\rho A}\right)}$ (C) $\sqrt{2\left(gh + \frac{mg}{A}\right)}$ (D) $\sqrt{2gh + \frac{mg}{A}}$

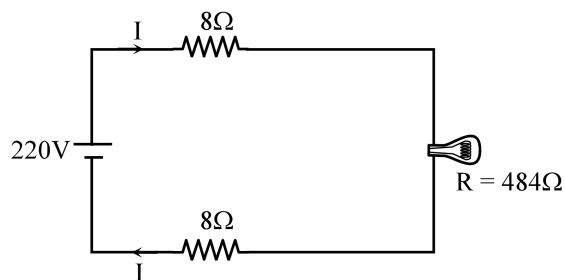
20. The kinetic energy of a particle along a circle of radius R is given as $K = As$ where A is a constant and s is the distance travelled. What is the net force acting on the particle -

- (A) $\frac{A}{R} \sqrt{1 + \frac{R^2}{s^2}}$ (B) $A \sqrt{1 + \frac{4s^2}{R^2}}$ (C) $4As \sqrt{1 + \frac{s^2}{R^2}}$ (D) none of the above

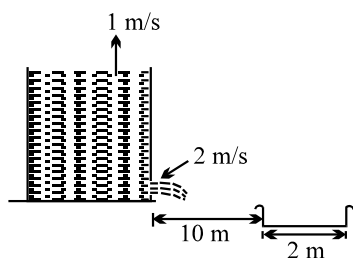
PART A: PHYSICS (SECTION B)

21.

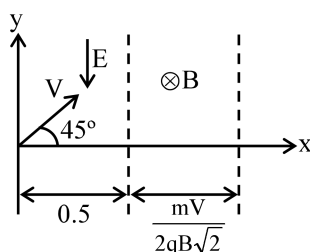
Using a long extension cord in which each conductor has a resistance 8Ω , a bulb marked as '100 W, 200 V' is connected to a 220 V dc supply of negligible internal resistance as shown in figure. Power delivered to the bulb is $\text{_____} \times 121 \times (0.44)^2$ watt.



22. A tank filled with water moves vertically with uniform velocity 1 m/s. There is a hole in its wall near bottom from which water comes out with constant relative velocity of 2 m/s in horizontal direction. If there is a tray which is 2m wide kept at 10 m from initial position of tank on floor find for what duration will the water be falling in the tray. ($g = 10 \text{ m/s}^2$) _____.



23. A charge particle of charge q and mass m is projected in a region which contains electric and magnetic field as shown in figure with velocity V at an angle 45° with x direction. If $V = \sqrt{\frac{qE}{m}}$, then net deviation in particle motion will be (neglect the effect of gravity) in clockwise direction approx in radian _____.



24. The intensity of solar radiation hitting the upper atmosphere of the Earth is measured to be 1326 W/m^2 . If the peak amplitude of the electric field vector of this wave is $E_0 = X \text{ V/m}$, calculate the integer value of X . (Take $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$ and speed of light $c = 3 \times 10^8 \text{ m/s}$)

25. A lead sphere is steadily sinking in glycerine whose viscosity is equal to $\eta = 13.9 \text{ P}$. What is the maximum diameter of the sphere at which the flow around that sphere still remains laminar? It is known that the transition to the turbulent flow correspond to Reynolds number $R_e = 0.5$ (Here the characteristic length is taken to be the sphere diameter). (in mm)

PART B: CHEMISTRY (SECTION A)

26. Which of the following is correct?

- (A) The value of steric factor P (regarding Arrhenius equation) decreases as the complexity of reacting molecules decreases.
- (B) During any individual reactive collision, the total energy of the colliding particles remains constant.
- (C) During a free radical chain reaction, if chain termination is faster than chain branching and chain propagation, explosion occurs.

(D) In a termolecular elementary process: $O + O_2 + N_2 \rightarrow O_3 + N_2$; N_2 serves as catalyst, while the order of reaction is two (2).

27. In which of the following M–F bond is strongest, among the given four compounds? (M = central atom)

- (A) CF_4 (B) NF_3 (C) BF_3 (D) OF_2

28. Which of the following is most stable?



29. What is the pH of a saturated aqueous solution of AgOH?

Given $K_{sp}(AgOH) = 1.5 \times 10^{-8}$, $\log(1.225) = 0.088$; $\log(1.5) = 0.176$

- (A) 8.176 (B) 7.824 (C) 4.088 (D) 10.088

30. Which of the following is correct?

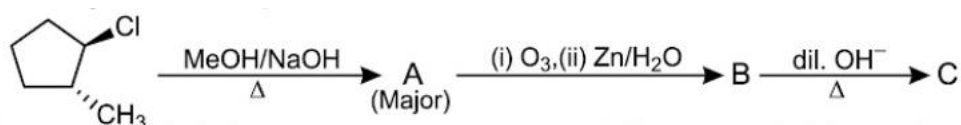
- (A) Kjeldahl's method of estimation of nitrogen is not applicable to the compounds containing nitrogen in nitro groups, azo groups and nitrogen present in the ring.
- (B) In Duma's method of estimation of nitrogen, the organic compound containing nitrogen is heated with Cu_2O in atmosphere of CO_2 to yield H_2O , CO_2 and free N_2 gas along with free copper.
- (C) During Carius method of estimation of halogen, a known mass of organic compound is heated with dil. HNO_3 , in presence of $AgNO_3$.
- (D) Phosphorous cannot be detected by Lassaigne's test.

31. Which of the following is not the constituent of 'Acid Rain'?

- (A) H_2CO_3 (B) H_2SO_4 (C) HNO_3 (D) H_3PO_4

32. Analyse the following reaction sequence and select the correct statement, following the reaction:

Cyclopentane derivative with Cl and CH_3 substituents



- (A) 'A' has 1-degree of unsaturation.

- (B) 'B' has 2-degree of unsaturation.
- (C) 'C' is an α, β -unsaturated ketone with 2-degree of unsaturation.
- (D) All other three statements are incorrect.

33. Which of the following give Glycine?

- (A) $\text{CH}_3\text{CH}_2\text{CH}=\text{CH}-\text{CH}_2-\text{CH}_3 \xrightarrow{\text{O}_3/\text{H}_2\text{O}} \xrightarrow{(i) \text{Br}_2/\text{red P} (ii) \text{H}_2\text{O}} \xrightarrow{\text{NH}_3 (\text{excess})} \xrightarrow{\text{H}_2\text{O}} ?$
- (B) $\text{CH}_3\text{CH}=\text{CHCH}_3 \xrightarrow{\text{O}_3/\text{Zn}} \xrightarrow{\text{NH}_4\text{Cl}/\text{NaCN}} \xrightarrow{\text{H}_3\text{O}^+/\Delta} \xrightarrow{\text{Very dil. OH}^-} ?$
- (C) $\text{CH}_3\text{CH}=\text{CHCH}_3 \xrightarrow{\text{O}_3/\text{H}_2\text{O}} \xrightarrow{\text{Br}_2/\text{red P}} \xrightarrow{\text{NH}_3 (\text{excess})} \xrightarrow{\text{H}_2\text{O}} ?$
- (D) $\text{CH}_3\text{COCO}_2\text{H} + \text{NH}_3 \xrightarrow{\text{H}_2/\text{Pd}} ?$

34. Which one of the following statements is correct?

(Please ignore the balancing wherever applicable)

- (A) $\text{Al} + \text{dil. H}_2\text{SO}_4 \rightarrow \text{Al}_2(\text{SO}_4)_3 + \text{SO}_2 + \text{H}_2\text{O}$
- (B) $\text{Al} + \text{AlCl}_3 \xrightarrow{\text{High temperature}} \text{Al}_2\text{Cl}_6$
- (C) $\text{FeSO}_4 \cdot (\text{NH}_4)_2\text{SO}_4 \cdot 6\text{H}_2\text{O}$ is called Mohr's salt
- (D) AlCl_3 is stable up to 800°C

35. Which of the following molecules is non-linear?

- (A) HCN (B) CO_2 (C) $[\text{AgCl}_2]^-$ (D) BaCl_2

36. Which of the following is expected to be most acidic solution?

- (A) Dilute aq. H_3BO_3 solution (B) Dilute aq. CH_3COOH solution
- (C) Dilute aq. HCOOH solution (D) Aq. $\text{Be}(\text{OH})_2$ solution

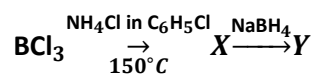
37. Which of the following process is expected to have the largest value of equilibrium constant, at room temperature?

- (A) $\text{H}_3\text{PO}_2 \rightleftharpoons \text{H}^+ + \text{HPO}_2^-$ (K₁) (B) $\text{H}_3\text{PO}_3 \rightleftharpoons \text{H}^+ + \text{H}_2\text{PO}_3^-$ (K₂)
- (C) $\text{H}_3\text{PO}_4 \rightleftharpoons \text{H}^+ + \text{H}_2\text{PO}_4^-$ (K₃) (D) $\text{H}_2\text{PO}_4^- \rightleftharpoons \text{H}^+ + \text{HPO}_4^{2-}$ (K₄)

38. The transition metal (M) complex that can have all isomers (geometric, linkage, and ionization) is:

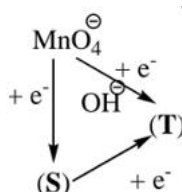
- (A) $[M(NH_3)_4Br_2]SCN$ (B) $[M(NH_3)_4Cl_2]Br$
 (C) $[M(NH_3)_4(H_2O)_2]Cl_3$ (D) $[M(NH_3)_4(H_2O)_2](SCN)$

39. The products X and Y in the following reaction sequence, respectively, are:



- (A) $B_3N_3Cl_6$ and $B_3N_3H_6$ (B) $B_3N_3H_3Cl_3$ and $B_3N_3H_6$
 (C) $B_3N_3H_3Cl_3$ and $B_3N_3H_{12}$ (D) $B_3N_3H_9Cl_3$ and $B_3N_3H_{12}$

40. Following is the reaction flow chart for manganese oxido complexes under different alkaline pH conditions. Compounds (S) and (T) respectively are:

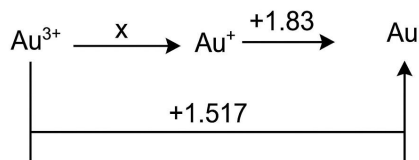


- (A) S = $MnO(OH)_2$; T = $Mn(OH)_2$ (B) S = MnO_2 ; T = $MnO(OH)$
 (C) S = MnO_4^{2-} ; T = $MnO(OH)$ (D) T = MnO_4^{2-} ; S = MnO_2

41. Number of different types of dipeptides produced using a mixture of glycine and L-valine, and number of optically active dipeptides formed in this mixture will be:

- (A) Four dipeptides, all optically active
 (B) Two dipeptides, all optically active
 (C) Four dipeptides, three optically active
 (D) Two dipeptides, none optically active

42. Latimer diagram are the compact representations of electrochemical equilibria in substances of multiple oxidation states. The value of the potential, x, in the Latimer diagram of gold (at pH = 1.0) is:



Given: $Au^{3+} \rightarrow Au$ has $E^\circ = +1.517 \text{ V}$

- (A) 2.72 V (B) 3.18 V (C) -3.18 V (D) 1.36 V

43. Choose the INCORRECT statement

- (A) Among the isotopes of carbon, ^{13}C is a radioactive isotope.

- (B) Carbon exhibits negative oxidation states along with +4 and +2.
- (C) Carbon cannot exceed its covalency more than four.
- (D) CO_2 is the most acidic oxide among the dioxides of group 14 elements.

44. Two liquids A and B form an ideal solution at temperature T K. At T K, the vapour pressures of pure A and B are 55 and 15 kN m^{-2} respectively. What is the mole fraction of A in solution of A and B in equilibrium with a vapour in which the mole fraction of A is 0.8?

- (A) 0.5217 (B) 0.480 (C) 0.663 (D) 0.340

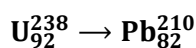
45. The wavelength of light absorbed for the following complexes are in the order:

$[\text{Co}(\text{NH}_3)_6]^{3+}$; $[\text{Co}(\text{H}_2\text{O})_6]^{3+}$; $[\text{Co}(\text{CN})_6]^{3-}$; $[\text{Co}(\text{NH}_3)_5(\text{H}_2\text{O})]^{3+}$; $[\text{CoF}_6]^{3-}$

- | | | | | |
|---------------------------|------|---------------------------|------|-----|
| (I) | (II) | (III) | (IV) | (V) |
| (A) III < I < II < IV < V | | (B) III < I < IV < V < II | | |
| (C) III < IV < I < II < V | | (D) III < I < IV < II < V | | |

PART B: CHEMISTRY (SECTION B)

46. The total number of α and β particles emitted in the following radioactive decay is x . Find the value of $\frac{5x}{11}$



47. For the elementary reaction: $C \xrightarrow{k_2} A \xrightarrow{k_1} B$, $k_1 = 2k_2$

At time $t = 0$, $[A] = A_0$ and $[B] = [C] = 0$. At a later time t , the value of $[B]/[C]$ is _____
(round off to the nearest integer)

48. The number of unpaired electrons in $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$ is _____

49. If the value of the equilibrium constant for the following equilibria is 10^{+x} , then find the value of x . $2\text{Fe}^{3+} + 2\text{I}^- \rightleftharpoons 2\text{Fe}^{2+} + \text{I}_2$

Given: $E_{\text{Fe}^{3+}/\text{Fe}^{2+}}^\circ = 0.77 \text{ V}$, $E_{\text{I}_2/\text{I}^-}^\circ = 0.53 \text{ V}$ $\frac{2.303RT}{F} = 0.06$

50. 18 gm of glucose ($\text{C}_6\text{H}_{12}\text{O}_6$) is treated with sufficient amount of HIO_4 under requisite conditions. How much formaldehyde (in gm) is formed?

PART C: MATHEMATICS (SECTION A)

51. The value of $\sin^{-1} \left\{ \cot \left(\sin^{-1} \sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1} \frac{\sqrt{12}}{4} + \sec^{-1} \sqrt{2} \right) \right\}$ is equal to:

- (A) 1 (B) 0 (C) -1 (D) 2

52. From a lot containing 20 defective and 80 non-defective bulbs, 6 bulbs are selected one by one with replacement. If the probability that exactly 5 bulbs are defective is P , then the value of $5^6 P$ is:

- (A) 24 (B) 30 (C) 48 (D) 60

53. Let $A = \{6, 10, 14, \dots, 1002\}$ and B is the set of divisors of the integer 360, then $S(A \cap B)$ is [$S(A)$ denotes sum of elements in set A]:

- (A) 148 (B) 154 (C) 156 (D) 162

54. Number of integral values of λ for which $f(x) = \sqrt{\ln(2\lambda \cos x + 5)}$ is defined for all $x \in \mathbb{R}$ is:

- (A) 5 (B) 4 (C) 6 (D) 11

55. The solution of the differential equation $\{y(1+x^{-1}) + \sin y\}dx + (x + \ln x + x \cos y)dy = 0$ is:

- (A) $x + y \ln x + y \sin x = C$ (B) $xy + y \ln x + x \sin y = C$
(C) $y + x \ln x + x \sin y = C$ (D) $xy + x \ln x + y \sin y = C$

(where C is arbitrary constant)

56. If in triangle ABC , $A = (1, 10)$, circumcentre $= \left(-\frac{1}{3}, \frac{2}{3}\right)$ and orthocenter $= \left(\frac{11}{3}, \frac{4}{3}\right)$, then the coordinates of mid-point of side opposite to A is:

- (A) $\left(1, -\frac{11}{3}\right)$ (B) $(1, 5)$ (C) $(1, -3)$ (D) $(1, 6)$

57. Consider a circle with centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and the parabola is:

- (A) $\left(\frac{p}{2}, p\right)$ (B) $\left(\frac{p}{2}, -\frac{p}{2}\right)$ (C) $\left(-\frac{p}{2}, p\right)$ (D) $\left(-\frac{p}{2}, -p\right)$

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^y + b^{-y})^2 & (b^y - b^{-y})^2 & 1 \\ (c^z + c^{-z})^2 & (c^z - c^{-z})^2 & 1 \end{vmatrix}$$

is equal to:

- (A) $a^x + b^y + c^z$ (B) $a^x b^{-y} c^{-z}$ (C) $a^{2x} b^{2y} c^{2z}$ (D) 0

59. The value of $\sum_{r=1}^{1024} [\log_2 r]$ is equal to, ($[\cdot]$ denotes the greatest integer function):

- (A) 8192 (B) 8204 (C) 8194 (D) none of these

60. Number of natural numbers $< 2 \cdot 10^4$ which can be formed with the digits 1, 2, 3 only is equal to:

- (A) $\frac{3^5 + 2 \cdot 3^4 - 3}{2}$ (B) $\frac{3^6 - 2 \cdot 3^4 + 3}{2}$ (C) $\frac{3^7 - 1}{2}$ (D) none of these

61. The value of $i \log(x - i) + i^2 \pi + i^3 \log(x + i) + i^4 (2 \tan^{-1} x)$, $x > 0$ (where $i = \sqrt{-1}$) is:

- (A) 0 (B) 1 (C) 2 (D) 3

62. The mean and variance of a data set of 8 observations are 12 and 5, respectively. If one observation x is replaced by y , the mean and variance become 13 and 6, respectively. Then the value of $x + y$ is:

- (A) 22 (B) 24 (C) 26 (D) 28

63. The number of real solutions of the equation: $|x + 2| + |x - 2| = |2x - 1| + 3$ is:

- (A) 1 (B) 2 (C) 3 (D) 4

64. If $f(x) = \frac{\cos x}{2 \sin x} + \frac{x}{\tan x} + \frac{1}{2x}$ then $\lim_{x \rightarrow 0} \frac{f'(x)}{x}$ is equal to:

- (A) -2 (B) 0 (C) 1 (D) 2

65. If $y = f(x)$ satisfies the differential equation $x \frac{dy}{dx} + y(2x + 1) = x e^x$ and $f(0) = 0$, then the number of solution(s) of $f(x) = \frac{2}{x}$ is equal to:

- (A) 0 (B) 1 (C) 2 (D) more than 2

66. Let a_1, a_2, a_3, a_4 be four terms in an A.P. such that each term and the common difference d are integers. If $a_1 + a_2 + a_3 + a_4 = 48$ and $a_1 a_2 a_3 a_4 + d^4 = 369$, then the largest term of the A.P. is:

- (A) 21 (B) 23 (C) 24 (D) 27

67. The sum of all values of α , for which the shortest distance between the lines $\frac{x+1}{\alpha} = \frac{y-1}{-1} = \frac{z-2}{-\alpha}$ and $\frac{x-1}{\alpha} = \frac{y+1}{2} = \frac{z-3}{2\alpha}$ is $\sqrt{3}$, is:

- (A) 4 (B) 6 (C) -6 (D) -4

68. Consider the function $f: (0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = |\ln x| - |x - 2|$. Let the following statements be given:

- (I) f is differentiable for all $x > 0, x \neq 1, 2$.
 (II) f is increasing in $(0, 1)$.
 (III) f is decreasing in $(2, \infty)$.

Then:

- (A) All (I), (II) and (III) are TRUE (B) Only (I) and (II) are TRUE
 (C) Only (II) and (III) are TRUE (D) Only (I) and (III) are TRUE

69. If the domain of the function $f(x) = \sin^{-1}\left(\frac{2}{x^2-2x-3}\right)$ is $(-\infty, \alpha] \cup [\beta, \gamma] \cup [\delta, \infty)$, then the value of $\alpha + \beta + \gamma + \delta$ is:

- (A) 0 (B) 2 (C) 4 (D) 6

70. If $f(x)$ satisfies the functional integral relation $f(x) = x + \int_0^1 (t + xe^t) f(t) dt$, then the value of $f(0) + f(1)$ is equal to:

- (A) $\frac{10-e}{e-2}$ (B) $\frac{8+e}{3-e}$ (C) $\frac{6-e}{e-1}$ (D) $\frac{12+e}{4-e}$

PART C: MATHEMATICS (SECTION B)

71. 5 boys (B_1, B_2, B_3, B_4, B_5) and 5 girls (G_1, G_2, G_3, G_4, G_5) are to be seated around a round table such that boy and girl sit alternately and B_i does not sit beside G_i for all $i \in \{1, 2, 3, 4, 5\}$. If the number of such arrangements is N , then the sum of digits of N is equal to _____.

72. A sample of 20 observations has mean of 50 and variance of 1, while a sample of 40 observations has mean of 50 and standard deviation 2. The 2 samples are combined to give a complete set of 60 observations with variance σ^2 . Then $3\sigma^2$ is equal to _____.

73. Let $f(x) = \int \frac{7x^8+9x^6}{(1+x^2+x^4)^2} dx$, $x > 0$, with the condition $\lim_{x \rightarrow 0} f(x) = 0$ and $f(1) = \frac{1}{2}$.

Let $A = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{2} & f'(1) & 1 \\ \alpha^2 & 3 & 1 \end{bmatrix}$ and $B = \text{adj}(\text{adj}(A))$. If $|B| = 64$, then the value of α^2 is _____.

74. Let $f(\alpha)$ denote the area of the region in the first quadrant bounded by $x = 0$, $x = 1$, $y^2 = x$ and $y = |\alpha x - 4| - |1 - \alpha x| + \alpha x^2$. Then the value of $f(0) + f(1)$ is _____.

75. Let (h, k) lie on the circle $C: x^2 + y^2 = 4$ and the point $(2h + 13k + 2)$ lie on an ellipse with eccentricity e . Then the value of $\frac{5}{e^2}$ is equal to _____.