

CLASS 9 | MATHEMATICS | ICSE-STYLE CHAPTER TEST PURE ARITHMETIC: RATIONAL AND IRRATIONAL NUMBERS

Maximum Marks: 80 Writing Time: 3 hours Reading Time: 15 minutes

Name: _____ Batch: _____ Date: _____

Instructions: Answer on separate sheets. Do not write during reading time. Attempt all of Section A and any four complete questions from Section B. Show essential working. Give exact answers unless stated otherwise. Use ruler and compass where instructed. Marks are in [].

SECTION A - 40 MARKS (Attempt all questions.)

Question 1. Choose the correct answer. Write the option and answer only. [15 x 1 = 15]

- (1) Which of the following is a rational number?
(a) $\sqrt{2}$ (b) $\sqrt{7}$ (c) $\sqrt{49}$ (d) π
- (2) The decimal expansion of an irrational number is:
(a) terminating (b) non-terminating recurring (c) non-terminating non-recurring (d) always negative
- (3) Which statement is true?
(a) every real number is rational (b) every integer is rational
(c) every irrational number is an integer (d) zero is irrational
- (4) The fraction $\frac{7}{40}$ has a decimal expansion which is:
(a) terminating (b) recurring (c) non-recurring and infinite (d) undefined
- (5) The simplest form of $\sqrt{72}$ is:
(a) $8\sqrt{2}$ (b) $6\sqrt{2}$ (c) $12\sqrt{6}$ (d) $36\sqrt{2}$
- (6) The value of $\sqrt{3} \times \sqrt{12}$ is:
(a) $\sqrt{15}$ (b) 3 (c) 6 (d) 12
- (7) Which number is irrational?
(a) $(\sqrt{5})^2$ (b) $\sqrt{18}/\sqrt{2}$ (c) $3 + \sqrt{2}$ (d) 0.333... (3 repeats)
- (8) The rationalising factor of $\sqrt{7} - \sqrt{3}$ is:
(a) $\sqrt{7} + \sqrt{3}$ (b) $\sqrt{7} - \sqrt{3}$ (c) $\sqrt{21}$ (d) 4
- (9) The rationalised form of $\frac{1}{\sqrt{5}}$ is:
(a) $\sqrt{5}$ (b) $\frac{\sqrt{5}}{5}$ (c) $5\sqrt{5}$ (d) $\frac{1}{5}$
- (10) Which number lies strictly between $\sqrt{2}$ and $\sqrt{3}$?
(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{7}{4}$
- (11) If $x = \sqrt{5} + 2$ and $y = \sqrt{5} - 2$, then xy equals:
(a) 9 (b) 5 (c) 1 (d) $\sqrt{5}$
- (12) Between any two distinct rational numbers, there are:
(a) no irrational numbers (b) exactly one irrational number
(c) finitely many irrational numbers (d) infinitely many irrational numbers
- (13) Statement I: The sum of two irrational numbers is always irrational. Statement II: A non-zero rational number multiplied by an irrational number is irrational.
(a) both true (b) both false (c) I true, II false (d) I false, II true
- (14) Assertion (A): $\sqrt{2} + \sqrt{8}$ is irrational. Reason (R): $\sqrt{8} = 2\sqrt{2}$.
Choose the correct relationship:
(a) A true, R false (b) A false, R true
(c) both true; R explains A (d) both true; R does not explain A
- (15) A student writes $\sqrt{9 + 16} = \sqrt{9} + \sqrt{16} = 7$. The correct conclusion is:
(a) the rule is valid (b) $\sqrt{25} = 5$; the rule used is invalid
(c) $\sqrt{25}$ is irrational (d) square roots cannot be evaluated

Question 2 [12]

- (i) Classify each as rational or irrational, giving a reason: $\sqrt{81}$, $\sqrt{11}$, 0.272727... (27 repeats), and 0.101001000100001... (successive blocks of zeros increase by one). [4]
- (ii) Express 0.454545... (45 repeats) as a fraction in lowest terms. Show the algebraic steps. [4]
- (iii) Simplify: (a) $\sqrt{48} + \sqrt{75} - \sqrt{27}$; (b) $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$. [4]

Question 3 [13]

- (i) Rationalise the denominator and simplify: $\frac{5}{(\sqrt{3} + \sqrt{2})}$. [4]
- (ii) Without using decimal approximations, arrange $\sqrt{5}$, 2 and $\sqrt{3}$ in ascending order. Justify the order by comparing their squares. [4]
- (iii) Using ruler and compass, construct the point representing $\sqrt{2}$ on a number line, taking 1 unit = 2 cm. Show the right-angled triangle, transfer the length to the line, and explain why the constructed length is $\sqrt{2}$ units. [5]

SECTION B - 40 MARKS (Attempt any FOUR questions.)

Each question carries 10 marks. Questions move from direct procedures to reasoning and application.

Question 4 [10]

- (i) Write three distinct rational numbers strictly between $\frac{2}{5}$ and $\frac{3}{5}$. Show a suitable common denominator. [3]
- (ii) Without long division, decide whether $\frac{13}{125}$ and $\frac{17}{30}$ have terminating or recurring decimal expansions. Justify using the prime factors of each denominator. [3]
- (iii) Simplify fully: (a) $3\sqrt{20} - 2\sqrt{45} + \sqrt{80}$; (b) $\sqrt{50}/\sqrt{2} + \sqrt{27}/\sqrt{3}$. [4]

Question 5 [10]

- (i) Express $1.233333\dots$ (only the digit 3 repeats) as a fraction in lowest terms. [3]
- (ii) Rationalise and simplify: $7/(2\sqrt{5})$. State whether the result is rational or irrational. [3]
- (iii) Evaluate $(\sqrt{6} + \sqrt{2})^2$ and $(\sqrt{6} - \sqrt{2})^2$. Hence find their sum and classify that sum as rational or irrational. [4]

Question 6 [10]

- (i) A square tile has area 18 cm^2 . Find its side and perimeter in simplest surd form. [3]
- (ii) A rectangle has length $(\sqrt{5} + 1) \text{ cm}$ and breadth $(\sqrt{5} - 1) \text{ cm}$. Find its area and perimeter. Which of these two quantities has a rational numerical value? [3]
- (iii) Simplify by rationalising both denominators:
 $1/(\sqrt{5} - \sqrt{3}) + 1/(\sqrt{5} + \sqrt{3})$. State whether the final answer is rational or irrational. [4]

Question 7 [10]

- (i) Find one rational and one irrational number strictly between 2 and 3. Justify that each lies in the interval without decimal approximations. [3]
- (ii) On a number line, points A and B represent $-\sqrt{3}$ and $\sqrt{12}$ respectively. Find the exact distance AB in simplest surd form and classify its numerical value. [3]
- (iii) Using ruler and compass, represent $\sqrt{5}$ on a number line (1 unit = 1 cm). Start with a right-angled triangle having perpendicular sides 2 cm and 1 cm. Show all construction arcs and explain your method. [4]

Question 8 [10]

- (i) A student claims, "The sum and the product of any two irrational numbers are always irrational." Disprove each claim using a separate pair of irrational numbers. Show the resulting sum and product. [3]
- (ii) Simplify $(\sqrt{3} + \sqrt{2})/(\sqrt{3} - \sqrt{2})$, expressing the result in the form $a + b\sqrt{6}$, where a and b are integers. State a and b. [3]
- (iii) Prove that $\sqrt{3}$ is irrational by assuming $\sqrt{3} = p/q$ in lowest terms and deriving a contradiction. Explain why both p and q would be divisible by 3. [4]

Question 9 [10]

- (i) If $x = 2 + \sqrt{3}$, show by rationalisation that $1/x = 2 - \sqrt{3}$. Hence find $x + 1/x$. [3]
- (ii) Using the result of part (i), find $x^2 + 1/x^2$ without separately expanding x^2 and $1/x^2$. [3]
- (iii) Prove that $5 - 2\sqrt{3}$ is irrational, using the fact that $\sqrt{3}$ is irrational. State explicitly which property of rational numbers gives the contradiction. [4]

Question 10 [10]

- (i) Simplify $\sqrt{9 + 4\sqrt{5}}$ by writing $9 + 4\sqrt{5}$ in the form $(\sqrt{a} + \sqrt{b})^2$, where a and b are positive integers. [3]
- (ii) Find rational numbers a and b such that $(\sqrt{5} + \sqrt{3})/(\sqrt{5} - \sqrt{3}) = a + b\sqrt{15}$. Show all working. [3]
- (iii) Prove that $\sqrt{2}$ is irrational. Hence prove that $1/(1 + \sqrt{2})$ is irrational by first rationalising its denominator. [4]