

CLASS 12 | MATHEMATICS (041) | CBSE-STYLE CHAPTER TEST

RELATIONS AND FUNCTIONS

Maximum Marks: 80 Writing Time: 3 hours Reading Time: 15 minutes

Name: _____ Batch: _____ Date: _____

Instructions: All 38 questions are compulsory. Choose one alternative where an internal choice is given. Show reasons and essential working. No calculators. $\mathbb{R}$ denotes real numbers; $\mathbb{Z}$ integers; $\mathbb{N} = \{1, 2, 3, \dots\}$. R denotes a relation. This chapter-only practice paper adapts the 2026-27 CBSE Mathematics format.

SECTION A - MCQs AND ASSERTION-REASON [20 x 1 = 20]

- A relation R on A is reflexive when:
(a) $(a, a) \in R$ for every $a \in A$ (b) R is empty (c) R has no diagonal pairs (d) A has one element
- On $A = \{1, 2\}$, which relation is the identity relation?
(a) $\{(1, 2), (2, 1)\}$ (b) $\{(1, 1), (2, 2)\}$ (c) $\{(1, 1)\}$ (d) $A \times A$
- A relation is an equivalence relation if it is:
(a) only reflexive (b) only symmetric (c) reflexive, symmetric and transitive (d) only transitive
- The empty relation on a non-empty set is:
(a) reflexive only (b) symmetric and transitive, not reflexive
(c) reflexive and symmetric (d) neither symmetric nor transitive
- The universal relation on a non-empty set is:
(a) an equivalence relation (b) never reflexive (c) never symmetric (d) never transitive
- A function $f: A \rightarrow B$ is onto precisely when:
(a) every input has two outputs (b) its range equals B
(c) all outputs are equal (d) distinct inputs always have distinct outputs
- The function $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x - 7$, is:
(a) one-one and onto (b) one-one, not onto (c) onto, not one-one (d) neither
- The function $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2$, is:
(a) one-one and onto (b) one-one, not onto (c) onto, not one-one (d) neither
- The function $f: [0, \infty) \rightarrow [0, \infty), f(x) = x^2$, is:
(a) one-one and onto (b) one-one only (c) onto only (d) neither
- For $f: \mathbb{N} \rightarrow \mathbb{N}, f(n) = n + 1$, the correct classification is:
(a) one-one and onto (b) one-one, not onto (c) onto, not one-one (d) neither
- On $\mathbb{Z}$, aRb means $a - b$ is even. Which pair belongs to R ?
(a) $(2, 5)$ (b) $(3, 8)$ (c) $(-1, 3)$ (d) $(0, 7)$
- On $\mathbb{R}$, the relation aRb defined by $a \leq b$ is:
(a) reflexive and symmetric only (b) reflexive and transitive, not symmetric
(c) symmetric and transitive only (d) an equivalence relation
- On $\mathbb{R}$, aRb means $|a| = |b|$. This relation is:
(a) only reflexive (b) only symmetric (c) an equivalence relation (d) not transitive
- If A has 3 elements, the number of reflexive relations on A is:
(a) 8 (b) 27 (c) 64 (d) 512
- For $A = \{1, 2, 3\}$, $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ is:
(a) not reflexive (b) an equivalence relation (c) not symmetric (d) not transitive
- Which function from $\mathbb{R}$ to $\mathbb{R}$ is onto but not one-one?
(a) $x \mapsto x^2$ (b) $x \mapsto 2x + 1$ (c) $x \mapsto x^3 - x$ (d) $x \mapsto |x|$
- For $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = kx + 5$, f is one-one exactly when:
(a) $k = 0$ (b) $k \neq 0$ (c) $k = 5$ only (d) $k > 5$ only
- If A and B each have 4 elements, every one-one function $A \rightarrow B$ must be:
(a) constant (b) onto (c) many-one (d) undefined at one input

For Questions 19-20: (a) A and R are true; R explains A . (b) A and R are true; R does not explain A . (c) A is true; R is false. (d) A is false; R is true.

19. Assertion (A): $f: \mathbb{R} \rightarrow [0, \infty), f(x) = x^2$, is onto. Reason (R): Every $y \geq 0$ has a real preimage $\sqrt{y}$.

20. A: Every symmetric relation is transitive. R: Symmetry requires $(b, a) \in R$ whenever $(a, b) \in R$.

SECTION B - VERY SHORT ANSWERS [5 x 2 = 10]

- On $A = \{1, 2, 3\}$, $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$. Is R reflexive? Is it symmetric? Justify both answers. [2]
- Test whether $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = 5x + 2$, is one-one. [2]
OR: Test whether $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2 + 1$, is one-one, giving a counterexample if needed. [2]
- For $f: \mathbb{Z} \rightarrow \mathbb{Z}, f(n) = 2n$, state its range and determine whether it is onto. [2]
- On $\mathbb{R}$, aRb means $a - b = 1$. Test reflexivity and symmetry with reasons. [2]
OR: On $\mathbb{R}$, aRb means $a + b = 0$. Test reflexivity and symmetry with reasons. [2]
- Give a relation on $\{1, 2\}$ that is reflexive and symmetric but not transitive, or explain why no such relation exists. [2]

SECTION C - SHORT ANSWERS [6 x 3 = 18]

26. On $\mathbb{Z}$, define aRb if $a - b$ is divisible by 3. Prove that R is an equivalence relation. [3]
27. Prove that $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 4x - 3$, is both one-one and onto. [3]
OR: Prove that $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^3$, is both one-one and onto. [3]
28. On $\mathbb{R}$, define aRb if $ab \geq 0$. Test reflexivity, symmetry and transitivity, giving a counterexample for any failed property. [3]
29. For $f: \mathbb{N} \rightarrow \mathbb{N}$, $f(n) = n^2$, test whether f is one-one and onto. Give reasons, including a codomain element with no preimage if appropriate. [3]
OR: For $f: \mathbb{Z} \rightarrow \mathbb{Z}$, $f(n) = n^2$, test whether f is one-one and onto, with supporting counterexamples where needed. [3]
30. On $A = \{1, 2, 3\}$, $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$. Test all three equivalence properties. For each failed property, identify missing pairs that show the failure. [3]
31. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be $f(x) = ax + b$. Determine the condition on a for f to be bijective, and prove your conclusion. [3]
OR: Show that $f: \mathbb{R} \rightarrow (0, \infty)$, $f(x) = 2^x$, is one-one and onto. [3]

SECTION D - LONG ANSWERS [4 x 5 = 20]

32. Let $A = \{1, 2, 3, 4, 5, 6\}$; aRb if a and b have the same parity. (a) Prove that R is an equivalence relation. [3] (b) List the elements related to 1 and those related to 2. Explain why these groups are disjoint and together cover A . [2]
33. Define $f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \setminus \{2\}$ by $f(x) = (2x + 3)/(x - 1)$. (a) Show that f is well-defined and one-one. [3] (b) Prove it is onto by finding an admissible input for every $y \neq 2$. [2]
OR: For $f: (-1, \infty) \rightarrow (0, \infty)$, $f(x) = 1/(x + 1)$, establish that f is well-defined, prove it is one-one, and prove it is onto by solving $f(x) = y$. [5]
34. On $\mathbb{Z}$, define aRb if $a^2 = b^2$. (a) Prove R is an equivalence relation. [3] (b) Find all integers related to 3 and all related to 0. Explain why 3 is not related to 4. [2]
35. Test $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2 - 4x + 5$, for one-one and onto properties. Identify its range. Then restrict its domain and codomain to make it bijective, and prove that your restrictions work. [5]
OR: For $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = |x - 2|$, establish its range and failure of one-one and onto properties. Restrict its domain and codomain to make it bijective and justify both properties. [5]

SECTION E - CASE STUDIES [3 x 4 = 12]

36. Activity groups: Six students have badge numbers $A = \{1, 2, 3, 4, 5, 6\}$. Two students are related when their badge numbers leave the same remainder on division by 3.
(i) List the badge numbers related to 1. [1] (ii) Are 2 and 5 related? Give a reason. [1] (iii) Prove the relation is symmetric and transitive. [2]
OR for (iii): List all ordered pairs in the relation and state whether it is reflexive. [2]
37. Locker allocation: Students $S = \{A, B, C, D, E\}$ are assigned lockers $L = \{1, 2, 3, 4\}$. The recorded allocation is $A \mapsto 1$, $B \mapsto 2$, $C \mapsto 3$, $D \mapsto 4$, $E \mapsto 4$.
(i) Does the allocation define a function $S \rightarrow L$? Explain. [1] (ii) Is it onto? Explain. [1] (iii) Is it one-one? Give evidence. Can any allocation of all five students to these four lockers be one-one? Explain. [2]
38. Temperature conversion: A sensor uses $F = f(C) = (9/5)C + 32$. Treat Celsius and Fahrenheit readings as real numbers, so $f: \mathbb{R} \rightarrow \mathbb{R}$.
(i) Find $f(0)$. [1] (ii) Find the input C when $F = 68$. [1] (iii) Prove that every real Fahrenheit reading has exactly one real Celsius preimage. Hence classify f . [2]
OR for (iii): If the input is restricted to $[-10, 40]$, give the exact codomain that makes f bijective and justify it. [2]